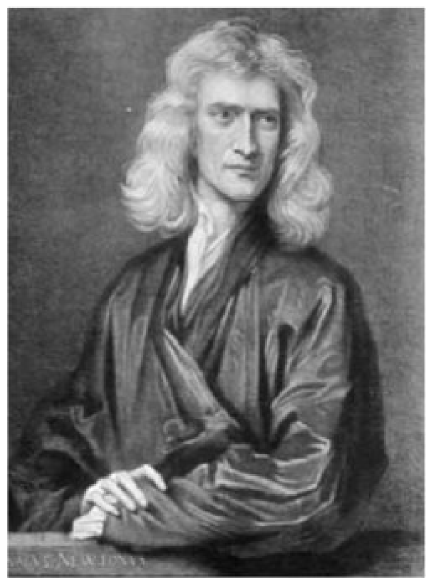
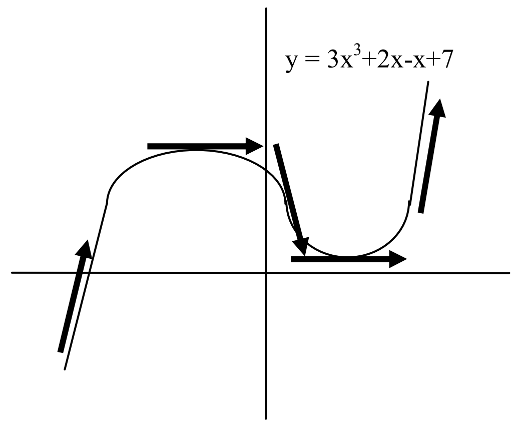
Physics 221
Calculus Physics Notes



Calculus Review

Basic knowledge of Calculus opens doors to a lot of problems in Physics and makes them much easier to solve. There are two things in Calculus that we need to know how to do; how to take derivatives and how to integrate. While you should find a good Calculus text for a full explanation, here are a couple of reminders.

The derivative express how a function is changing as we examine a small piece of it. Looking at the function to the right we can find places where it slopes down or slopes up or levels off flat. The arrows represent tangent lines, lines that only touch the curve at one point. If we want to know how something is changing, we look at the slope of the tangent line.



To find this slope we take the derivative. The derivative of a power of x is easy to find since

$$\frac{d}{dx}\left(Ax^{n}\right) = Anx^{n-1}$$

For a polynomial, simply take the derivative of each term as shown in the following example

$$\frac{d}{dx}\left(3x^{3} + 2x^{2} - x + 7\right)$$
$$= \frac{d}{dx}\left(3x^{3} + 2x^{2} - 1x^{1} + 7x^{0}\right)$$
$$= 3\cdot 3x^{3-1} + 2\cdot 2x^{2-1} - 1\cdot 1x^{1-1} + 7\cdot 0x^{0-1}$$
$$= 9x^{2} + 4x - 1x^{0} + 0$$
$$= 9x^{2} + 4x - 1$$

Now at any value of x we can find the slope of this function and see it on the graph above. If x = -3 in the example, then plug -3 in for x into the derivative $9(-3)^2+4(-3)-1 = 68$. Since the answer is positive and represents the slope, we know the curve heads up at x = -3. At x = -1, $9(-1)^2+4(-1)-1 = 4$ so the curve is still headed up, but not nearly so steeply. At x = 0, $9(0)^2+4(0)-1 = -1$, the slope is now negative and the curve heads down. Somewhere in between x = -1 and x = 0 the slope went to zero, which would be represented on the graph by a flat horizontal tangent line.

The easy way to find the point where the slope is zero is to set the derivative of the function equal to zero and solve for x. We solve $9x^2+4x-1 = 0$ by using the quadratic formula with a = 9, b = 4, c = -1

$$x = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^{2} - 4(9)(-1)}}{2(9)} \approx 0.178 \; or \; -0.623 \,.$$

These are the x values of the points where the slope = 0. At x = - 0.623 we see have a local maximum and at x = 0.178 we have a local minimum.

Other functions are trickier, but can usually be looked up in a table like the few shown here. The variables u and v represent functions of x.

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^{2}}$$

$$\frac{d}{dx}\left(e^{x}\right) = e^{x} \qquad \frac{d}{dx}\left(e^{u}\right) = e^{u}\frac{du}{dx}$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \qquad \frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\sin u) = \cos u\frac{du}{dx}$$

$$\frac{d}{dx}(\cos x) = -\sin x \qquad \frac{d}{dx}(\cos u) = -\sin u\frac{du}{dx}$$

$$\frac{d}{dx}(\tan x) = \sec^{2} x \qquad \frac{d}{dx}(\tan u) = \sec^{2} u\frac{du}{dx}$$

Before we saw that for polynomials

$$\frac{d}{dx}\left(3x^3 + 2x^2 - x + 7\right) = 9x^2 + 4x - 1$$

Here we had to multiply the coefficient by the exponent and lower the exponent by one. To get back to the original function we would have to do the opposite; raise the exponent by one and divide by the new exponent.

The integral is sometimes called the anti-derivative because it "undoes" the derivative. For our example we would use the following notation

$\int (9x^2 + 4x - 1)dx$ which is read "taking the integral of $9x^2+4x-1$ with respect to x." The dx notation simply tells us which variable to integrate. Here is the integration

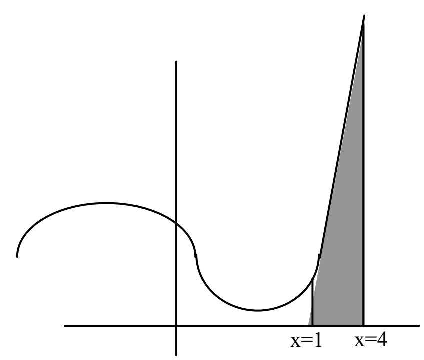
$$\int (9x^2 + 4x - 1)dx$$
$$= \frac{9x^{2+1}}{2+1} + \frac{4x^{1+1}}{1+1} - \frac{1x^{0+1}}{0+1}$$
$$= \frac{9x^3}{3} + \frac{4x^2}{2} - \frac{x^1}{1}$$
$$= 3x^3 + 2x^2 - x$$

Which is very close to the original $3x^3+2x^2-x+7$. However, we lost the constant term "+7" at the end, so whenever we integrate we can do one of two things; add a constant or use limits of integration.

Adding a constant is easy, $\int (9x^2 + 4x - 1)dx = 3x^3 + 2x^2 - x + C$. We don't what C is, but it represents that we recognize that there could have been something there before we took the derivative. Sometimes the problem gives enough information to find C.

If the problem specifies a range of x values for the problem then we use these as limits of integration. If we are interested in the region between x=1 and x=4 for $9x^2+4x-1$, then the integration is completed when we substitute these x values in. The integral is the (result with the upper limit) – (the result with the lower limit) as shown

$$\int_{x=1}^{x=4} (9x^2 + 4x - 1)dx$$
$$= 3x^3 + 2x^2 - x\Big|_{x=1}^{x=4}$$
$$= \left[3(4)^3 + 2(4)^2 - (4)\right] - \left[3(1)^3 + 2(1)^2 - (1)\right]$$
$$= [220] - [4]$$
$$= 216$$



This number is the area under the curve and between x = 1 and x=4. In the graph at the right, this area is shaded. There are many applications for finding the area under a curve in Physics and Engineering and Economics and many other fields.

Again if the function you wish to integrate is not a polynomial, there are tables to assist in finding the integrals. Here are a couple, but there are easily 500 or more in complete tables.

$$\int uv\,dx = uv - \int v\,du$$

$$\int e^{ax}dx = \frac{1}{a}e^{ax} \qquad \int xe^{ax}dx = \frac{e^{ax}}{a^2}\left(ax - 1\right) \qquad \int \ln ax\,dx = (x \ln ax) - x$$

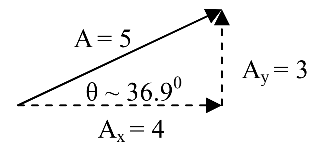
$$\int \sin ax\,dx = -\frac{1}{a}\cos ax \qquad \int \cos ax\,dx = \frac{1}{a}\sin ax \qquad \int \tan ax\,dx = -\frac{1}{a}\ln\left(\cos ax\right)$$

Test 1 - Measurement Topics

The extra topics included in Test 1 include knowing rectangular and polar notations for vectors and a couple of extra operations with vectors. Bold print denotes a vector.

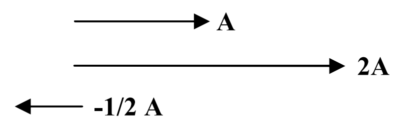
- **Use unit vector notation to describe vectors and vector components.**

  Vectors can be expressed using rectangular coordinates or polar coordinates. For a given vector, rectangular coordinates specify the x and y components, whereas polar coordinates give the magnitude and direction. In the example at the right, we can either write **A** = 4**i** + 3**j** or **A** = $5\angle 36.9^0$ where the first uses a rectangular coordinate system and the second is in polar coordinates. **i** and **j** are called unit vectors; they are 1 unit in length and point in the x and y direction respectively.



- **Multiply a vector by a scalar.**

  Multiplication by a scalar involves changing the magnitude of a vector. It can also reverse the direction of the vector. Simply multiply the vector by a number (also known as a scalar)



  For some vector R multiplied by the scalar 2, in rectangular and polar coordinates

  R = 4**i** + 3**j**   **R** = $5\angle 36.9^0$
  2 **R** = 2(4**i** + 3**j**)   2**R** = $2\cdot 5\angle 36.9^0$
  2 **R** = 8**i** + 6**j**   2**R** = $10\angle 36.9^0$

  Here is an example using a negative scalar. When using polar coordinates, simply add or subtract 180^0 to the angle to reverse the direction.

  R = 4**i** + 3**j**   **R** = $5\angle 36.9^0$
  -1.7 **R** = -1.7 (4**i** + 3**j**)   -1.7 **R** = $1.7(5)\angle(36.9^0+180^0)$
  -1.7 **R** = -6.8**i** - 5.1**j**   -1.7 **R** = $8.5\angle 216.9^0$

- **Obtain the dot product of two vectors**

  The dot product is defined

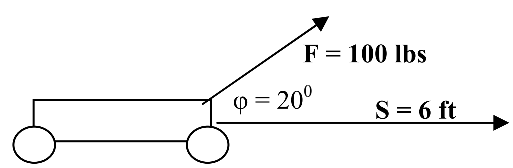
  Polar Coordinates: **A·B** = AB cos φ where φ is the angle between **A** and **B** **or**
  Rectangular Coordinates: **A·B =** $a_1b_1 + a_2b_2 + \ldots$

  Example: If **A** = $10\angle 23^0$ and **B** = $7\angle 82^0$ then **A·B** = 10·7 cos 59^0 = 70 (.5150) = 36.05
  If **A** = 3**i** + 5**j** and **B** = 6**i** + 12**j** then **A·B =** 3·6 + 5·12 = 78

  Note that the dot product results in a scalar number. For this reason, the dot product is also sometimes called the scalar product.

  The picture of the wagon shows an application of the dot product. A force **F** pulls up and to the right to move the wagon to the right. The upward force does nothing to move the wagon forward. To calculate the work performed,

  W = **F·s =** Fs cos φ = 100·6 cos 20^0 = 564 J

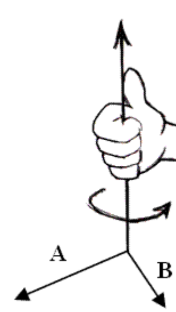


- **Obtain the cross product of two vectors.**

  Whereas the dot product results in a scalar, the cross product "multiplies" two vectors and **results in a third vector**. The magnitude of the cross product is given by

  |**AxB**| = AB sin φ   where φ is the angle between **A** and **B**

  and the direction is given by the "right hand rule". To use the right hand rule, place the fingers of your right hand in the direction of the first vector, curl them in the direction of



the second vector, your thumb will be pointing in the direction of the resulting vector.
Example: If **A** = $10\angle 23^0$ and **B** = $7\angle 82^0$ where A and B are both on the x-y plane, then
|**AxB**| = $10 \cdot 7 \sin 59^0 = 70\ (.8572) = 60.00$ and the direction is in the positive z direction.

Since the cross product results in another vector, sometimes it is called the vector product.

- **Practice Problems**

  A = 5**i** - 3**j**   **B** = 4**i** + 2**j**   **C** = $15\angle 243^0$ **D** = $12\angle 73^0$

  1) Express **A** in polar notation.
  2) Express **D** in rectangular notation.
  3) Find 2**B**
  4) Find 5**A – 2B**
  5) Find **A·B**
  6) Find **C·D**
  7) Does **A·B = B·A?** Show your reasoning.
  8) Find **AxB**
  9) Find **CxD**
  10) Does **AxB = BxA?** Show your reasoning.

Test 2 – Motion Topics

- **Determine velocity and displacement of freely falling objects.**
  We do this in the text book with the three formulas $v = v_0 + at$, $s = v_0t + ½ at^2$ and $v^2 = v_0^2 + 2as$. Here, we will do some extra problems for practice that are a step above those found in the text book.

- **Use position vectors in determining average and instantaneous velocity, and average and instantaneous acceleration.**
  The definitions in the text book can now be extended to use calculus,

$$v = \frac{\Delta s}{\Delta t} \quad so \;\; v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

$$a = \frac{\Delta v}{\Delta t} \quad so \;\; a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

Now, if given an expression for the position of an object, we can find the velocity and acceleration of the object. For example, if $s = 3t^2 – 4t + 43$, then

$$v = \frac{ds}{dt} = \frac{d}{dt}(3t^2 - 4t + 43) = 6t - 4$$

$$a = \frac{dv}{dt} = \frac{d}{dt}(6t - 4) = 6$$

If we wanted to know the speed and acceleration of this object after t = 17 s

$$v = 6(17) - 4 = 98$$

$$a = 6$$

We can also work the other way and solve these differential equations. If we know the acceleration of an object, we can find the velocity and position of the object as shown.

Since $a = \frac{dv}{dt}$

$a\,dt = dv$

$\int a\,dt = \int dv$

$at + c = v$

which is the familiar $v = v_0 + at$.

Then, since $v = \frac{ds}{dt}$

$v\,dt = ds$

$\int v\,dt = \int ds$

$\int (v_0 + at)\,dt = \int ds$

$v_0 t + \frac{1}{2}at^2 + c = s$

which is the familiar $s = s_0 + v_0 t + \frac{1}{2}at^2$

Another example: Find the velocity and position of a rocket after 12 seconds if acceleration is given by $a= 3t^2$ and we know that v = 200 ft at t = 4 s and s = 40 at t= 0 s.

$v = \int a\,dt = \int 3t^2\,dt = t^3 + C_1$

$Since\, v = t^3 + C_1\ and\ v = 200\ when\, t = 4$

$200 = 4^3 + C_1\ so\ C_1 = 136$

$Thus\ v = t^3 + 136$

$s = \int v\,dt = \int (t^3 + 136)dt = \frac{1}{4}t^4 + 136t + C_2$

$Since\, s = \frac{1}{4}t^4 + 136t + C_2\ and\ s = 40\ when\, t = 0$

$40 = \frac{1}{4}(0)^4 + 136(0) + C_2\ so\ C_2 = 40$

$Thus\ s = \frac{1}{4}t^4 + 136t + 40$

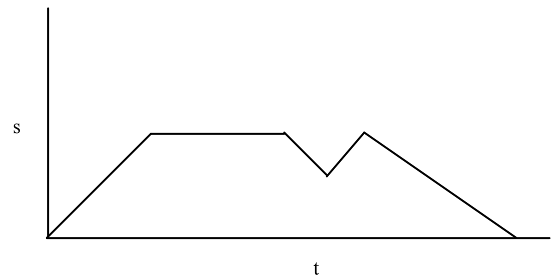
$At\, t = 12s,$

$v = (12)^3 + 136 = 1864\ ft/s$

$s = \frac{1}{4}(12)^4 + 136(12) + 40 = 6856\ ft$

In summary, $v = \frac{ds}{dt} \qquad a = \frac{dv}{dt} \qquad v = \int a\,dt \qquad s = \int v\,dt$

- **Practice Problems**
  1. **1)** Find the velocity and acceleration of a rocket 20 seconds after launch if it's position is given by $y = 3t^2 + 42t + 30$.
  2. **2)** Find the position, velocity and acceleration of a rocket car is traveling after 30s if it can accelerate at $a = 2 + 4t$ m/s^2, and is traveling at 16 m/s at t=1 s and is at s = 0 at t = 0.
  3. **3)** A speeder passes a cop in a speed trap. The cop is initially at rest but begins to accelerate at 2 m/s^2 at the instant the speeder passes. The speeder is unaware of the cop and continues at constant velocity until the cop catches up after 600 m. Find the velocity of the speeder.
  4. **4)** Little Johnny rides his bike from his house to the store. He looks at the comic books for a while and starts to ride home again, but then remembers his momma asked him to get some baking soda. He returns to the store, gets the soda and rides back home. A graph of his displacement from home in time would look like that below. Draw the graph for his velocity.



  5. **5)** In 1971 DB Cooper hijacked a 727 airplane, got \$200,000 from the airline company and parachuted from the back of the airliner at 10,000 ft into the stormy darkness of Thanksgiving eve night. He was never found or caught. Find estimates for the minimum horizontal drift distance (if his parachute didn't open) and the maximum horizontal drift distance (if his parachute did open). The plane was moving at 170 knots. Look up and use terminal velocity for a falling body and for a parachutist in both estimates. (just do your best – the FBI can't figure it out either!)
  6. **6)** The position of a particle moving along the x-axis is given by $x = 6 + 11\,t - 2t^2$ where x is in meters and t is in seconds.
     a) What is the velocity at t = 3s?
     b) Is the velocity constant, or is it constantly changing? Explain why.
     c) What is the acceleration at t = 0.5 s?
     d) Is the acceleration constant, or is it constantly changing? Explain why.
     e) In the expression above what are the units of 6, 11, and -2?
     f) In what direction is the velocity at t = 3 s?
     g) In what direction is the acceleration at t = 1 s?
  7. **7)** The position of a particle moving along the x-axis is given by $x = 3 - 27\,t + t^3$ where x is in meters and t is in seconds.
     a) Find the particles velocity function v(t) in m/s.
     b) Where is the particle when t = 0 s? t = 1 s?
     c) Find the particle's velocity (magnitude and direction) at t = 2 s and t = 4 s.
     d) Find the expression for the acceleration of the particle.
     e) Is there ever a time at which v = 0?
     f) Describe the particles motion x(t), v(t), a(t) for $t \geq 0$.
  8. **8)** The position of a particle moving along the x-axis is given by $x(t) = b + ct – dt^2 + et^3$ where x is in meters and t is in seconds.
     a) Find the function for v(t) in m/s.
     b) Find the function for a(t) in m/s^2.
     c) What are the units of b, c, d and e?
     d) Describe the particles motion x(t), v(t), a(t) for $t \geq 0$.
  9. **9)** A cockroach runs across a kitchen wall on which a set of coordinates has been drawn. The coordinates of the roach 's position are given by
     $x(t) = -2t^2 + 5t + 2$ and $y(t) = t^2 - 3t + 14$
     a) At t = 1 s, what is the insect's position vector in unit vector notation $\left(\vec{r} = a\hat{i} + b\hat{j}\right)$ and magnitude/angle notation $\left(\vec{r} = r\angle\theta\right)$
     b) Find the velocity function in unit vector notation and magnitude/angle notation $\left(\vec{v} = a\hat{i} + b\hat{j} = r\angle\theta\right)$.
     c) Find the acceleration function in unit vector notation and magnitude/angle notation $\left(\vec{a} = a\hat{i} + b\hat{j} = r\angle\theta\right)$.

10) The coordinates of a particle's position are given by $x(t) = -3t^3 - 4t$ and $y(t) = -5t^2 + 6$. Are the x and y components of the acceleration constant?

11) A particle's position vector is $\vec{r} = (4t^3 - 2t)\hat{i} + 3\hat{j}$ Is the acceleration constant.

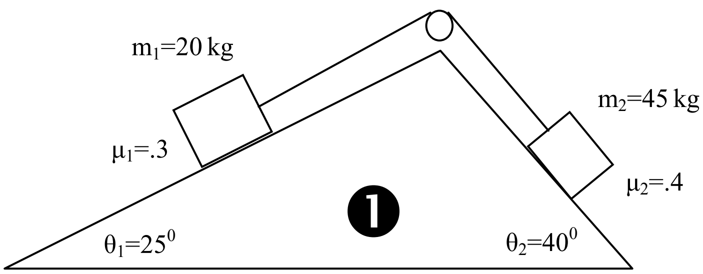
12) The acceleration of a moving object is given by a = 6t +2, with a in m/s^2 and t in seconds. At time t=0 the velocity was 8 m/a and the object's position was $s_0 = 0$. Find expressions for the following:

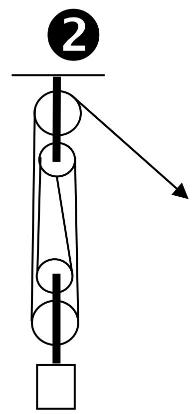
a) Velocity as a function of time
b) Position as a function of time
c) Calcul;ate the velocity at t = 1s, 3s, 5.5 s
d) How far has the object travelled after 1s and 4s

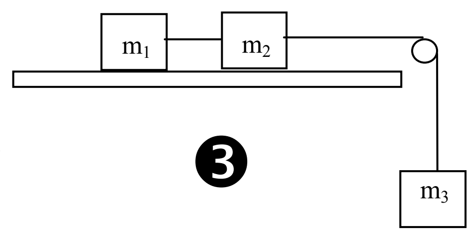
Test 3 - Force and Motion Topics

No new topics this time, just some problems to work.

- **Practice Problems**
    - **1)** Two masses are suspended by a string over a pulley as shown❶. Find the acceleration and tension in the string. Assume m_1 moves uphill.
    - **2)** A pulley system like the one shown❷ lifts 200 kg. Find the applied force (how hard you have to pull on the rope). Hint: think about the forces on the bottom pulley block.
    - **3)** Three blocks are moving together as shown❸. Find the hanging mass needed to accelerate the system at 1.2 m/s^2. m_1 = 6 kg, m_2 = 10 kg, μ = .9
    - **4)** If μ = .7, m_1 = 8 kg, m_2 = 12 kg and m_3 = 22 kg as shown ❸, find the acceleration, the tension between m_1 and m_2 and the tension between m_2 and m_3.







Test 4 - Energy, Work and Power Topics

- **Determine work done by a general variable force.**
  In the book, we calculate work with the formula W = Fs. This works great with a constant force. When the force is changing we have to add up the individual pieces of force times distance, and if the force is a continuous function then integration is our best bet. Thus,

  $$W = \int_{x_1}^{x_2} F\,dx$$

- **Determine work done by a spring force.**
  The best example of a variable force doing work is a spring. The spring force is given by F = -kx, that is, a bigger displacement causes a bigger restoring force. Since the force is not constant, we can't use W = Fx. The proper calculation of work requires the following, which is what the text book shows for work or energy of a spring:

$$W = \int F\,dx$$
$$W = \int (kx)\,dx$$
$$W = \frac{1}{2}kx^2$$

- **Obtain average and instantaneous power and know units for describing power.**
  We have seen that average power is given by P = W/t. As we look at a smaller and smaller slice of time, the instantaneous power is given by
  $$P = \frac{W}{t} \quad so \;\; P_{inst} = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$
  For example, to find the instantaneous power after 13s, if $W = 3t^2 + 2$
  $$then \;\; P_{inst} = \frac{d}{dt}(3t^2 + 2) = 6t = 6(13s) = 78W$$

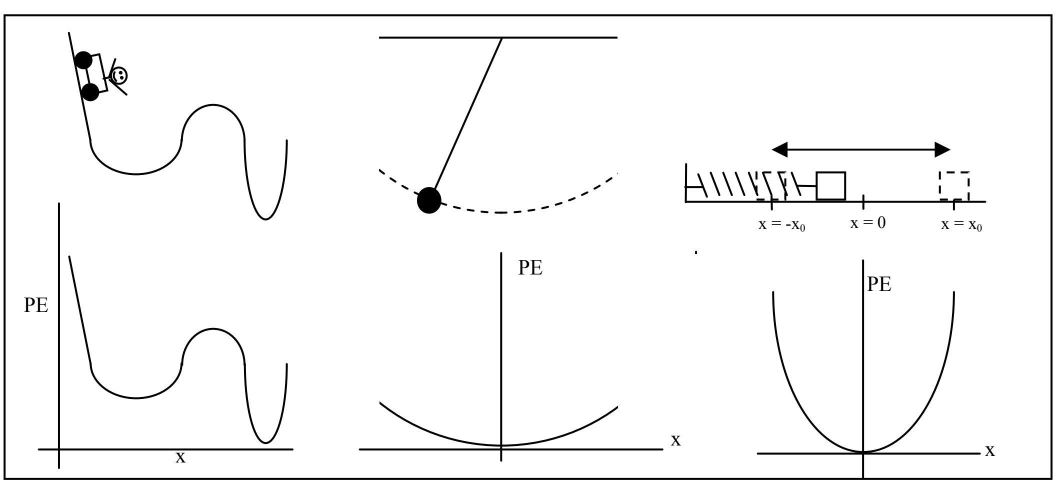
- **Demonstrate an understanding of and apply relationships involving potential energy and work.**
  We do this in the text book.

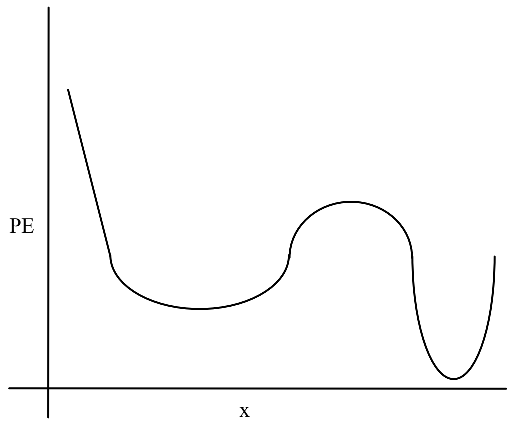
- **Use the concept of path independence in the solution of problems involving work.**
  Work is calculated by multiplying the applied force by the net displacement, W = Fs. If I use a force of 30 N to move an object .1 m then the work is calculated to be (30 N)(.1 m) = 3 J. If I use 30 N to move the object all over the room, down the hall, upstairs and then bring it back to set it down .1 m from its original position then displacement is still .1 m and work is still 3 J. This is the idea of path independence. The calculation of work only considers the beginning and final position of the object, not the path it took to get there.

  If the object ends up in its initial position, displacement = 0 and work = 0, meaning there was no net change – nothing happened. Another way to think of this seeming paradox of doing so much "work" to accomplish nothing is to count the work on the way there as positive work since the displacement would be a positive number. On the way back, we move the opposite (or negative) direction so that would be negative work.

- **Determine gravitational and elastic potential energy of a system.**
  Since work and energy are often different aspects of the same thing, the gravitational and potential energy are determined as follows:
  $$PE_{gravity} = \int F_{gravity}\,dy = \int_{y=0}^{h} (mg)\,dy = mgy\Big|_{y=0}^{h} = mgh$$
  $$PE_{elastic} = \int F_{elastic}\,dx = \int (kx)\,dx = \frac{1}{2}kx^2$$

- **Interpret potential energy curves for a system, and locate equilibrium points and turning points for a system.**
  Some potential energy curves (or graphs) are shown below. In the first example, a roller coaster loses PE (and develops KE) as it drops and PE increases when it goes uphill (at the expense of KE). The pendulum is constantly losing and gaining PE as it swings back and forth. Similarly, an object attached to a spring is continually trading spring PE for KE and back again.





- **Practice Problems**
  1) In a tractor pull, tractors pull a sled with a moving weight that transfers weight from the wheels to a skidpan, making it harder to pull. If the force needed to pull the sled increases with distance as $F = 170x + 9000$ lbs, find the work to pull the sled for the full 300 ft run.
  2) Find the speed of a 60 g dart shot from a dartgun with a spring constant of 500 N/m after being compressed .2 m.
  3) A compound bow will shoot a 118 g arrow at 55 m/s, a) how high would the arrow go if pointed straight up (don't try this!) b) If the string does work on the arrow for 40 cm, find the average force of the bow.
  4) A 16 g bullet moving at 366 m/s strikes a board and exits the other side at 120 m/s. Find the heat generated in the board if all of the lost energy of the bullet goes into heat.

Test 5 - Linear Momentum Topics

No new topics this time, just some problems to work.

- **Practice Problems**
  1) A 7 g bullet moving 500 m/s hits a 3 kg block of wood which is at rest, passes through it and comes out the other side moving 430 m/s. How fast is the block moving when the bullet emerges?
  2) A 150 kg astronaut is stuck motionless in space when his rocketpack malfunctions. His tool belt has a five kg drill motor, a four kg camera and a two kg hammer. If he can throw the objects at 6 m/s, find the speed he will move in the opposite direction if he throws **a)** the drill, **b)** the camera and **c)** the hammer.
  3) A 180 lb diver jumps into a pool of water from a height of 12 ft. If it takes half a second for him to come to a stop, what average force does the water put on him?

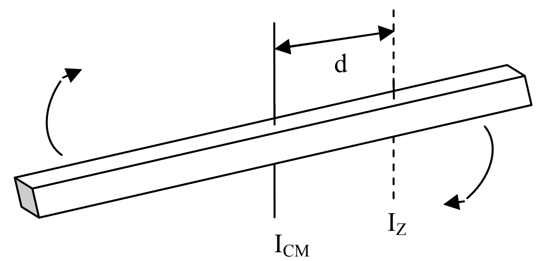
Test 6 - Rotational Motion Topics

- **Apply the parallel-axis theorem to determine moment of inertia of a body.**
  The text gives a number of formulas to use for some simple shapes when calculating the moment of inertia of everyday objects, each with an axis of rotation. To help match more real world cases that may not be as straightforward as the given cases, you can shift the location of the axis of rotation parallel to itself using the parallel axis theorem

  $$I_Z = I_{CM} + Md^2$$

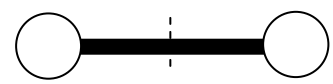
  For example, a stick rotated about the center has a moment of inertia of $I = ML^2/12$. Shifting the axis of rotation from the solid line through the center of the stick to the dotted line a distance d away is as simple as the following:

  $$I_Z = I_{CM} + Md^2$$
  $$= \frac{ML^2}{12} + Md^2$$



- **Demonstrate an understanding of and apply the Law of Conservation of Angular Momentum to problems involving systems of particles.**
  See the other book.

- **Practice Problems**
  1) Find the moment of inertia of a 5 kg sphere with a radius of 10 cm, rotated about the edge.
  2) Two 3 kg solid balls with a radius of 7 cm are connected by a 4 kg rod that is 2 m long. Find the torque necessary to accelerate the assembly at 5 rad/s^2 about the dotted line axis.



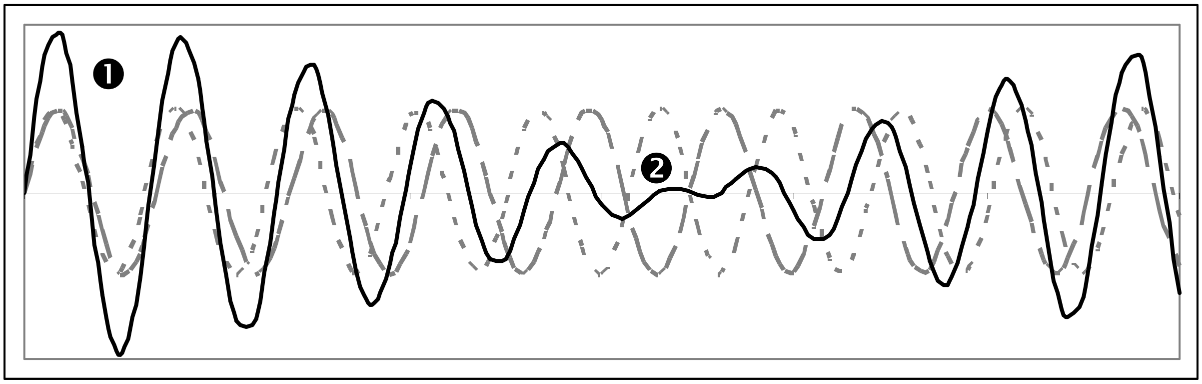
  3) A 60 kg girl is spinning on the edge of a 400 kg playground merry-go round with a radius of 2m at an angular velocity of 8 rad/s. She then moves to the middle of the merry-go-round. Find the new angular velocity.

Test 7 - Periodic Motion Topics

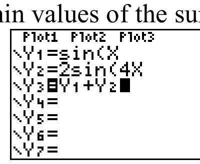
- **Describe three basic types of waves.**
  In **transverse waves** particles are vibrated up and down as the wave moves horizontally. Light and other forms of electro-magnetic radiation are examples of transverse waves.
  In **longitudinal waves** particles are pushed forward and back in the same direction as the wave motion. Sound waves push air particles (and ear drums) forward and back in low and high pressure cycles.
  Mixed waves have both types of motion above. An object in the water will bob up and down and move forward and back at the same time. The resulting particle motion is circular or elliptical.

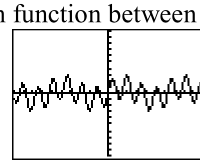
- **Define the Principle of Superposition for Waves**

When two or more waves occupy the same space, the wave forms add either constructively or destructively. Constructive interference, as it is called, occurs when two waves add to something bigger than either of the individual waves like shown below at ❶. The two dotted gray waves are passing through the same space and their sum, represented by the black line has a greater amplitude than either of the original waves. Destructive interference is shown at ❷, where the two waves cancel each other out.



- **Practice Problems**
  1) Derive the formulas $v = 2\pi f A\cos(2\pi f t)$ and $a = -4\pi^2 f^2 A\sin(2\pi f t)$ from $s = A\sin(2\pi f t)$.
  2) Use a graphing calculator to set up the following two sin functions and the sum of the two and graph just the sum. Find the max and min values of the sum function between x = 6 and x = 9.





Hints: a) set calc to Radian mode,
b) arrow back to the equal sign in "Y_1" and "Y_2" and hit "Enter" to clear the block so that the first two lines don't graph
c) for the third line, hit the "VARS" button, then right arrow, then Function, and then choose Y_1 or Y_2

Summary of New Stuff

- Use unit vector notation to describe vectors and vector components.
  A = 4**i** + 3**j** or **A** = $5\angle 36.9^0$

- Multiply a vector by a scalar.
  R = 4**i** + 3**j** **so** 2**R** = 2(4**i** + 3**j**) = 8**i** + 6**j**
  R = $5\angle 36.9^0$ **so** 2**R** = $2\cdot 5\angle 36.9^0 = 10\angle 36.9^0$

- Obtain the dot product of two vectors.
  A·B = AB cos φ where φ is the angle between **A** and **B**

- Obtain the cross product of two vectors.
  |**AxB|** = AB sin φ where φ is the angle between **A** and **B,** direction is given by the "right hand rule".

- Use position vectors in determining average and instantaneous velocity, and average and instantaneous acceleration.
  $$v = \frac{ds}{dt} \qquad a = \frac{dv}{dt} \qquad v = \int a\,dt \qquad s = \int v\,dt$$

- Determine work done by a general variable force.
  $$W = \int_{x_1}^{x_2} F\,dx$$

- Apply the parallel-axis theorem to determine moment of inertia of a body.
  $$I_Z = I_{CM} + Md^2$$