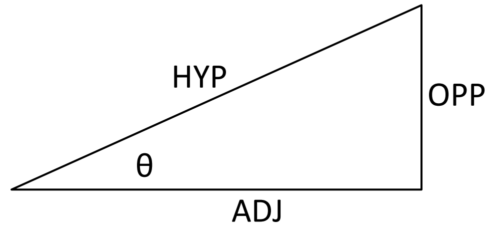
Right Triangle Trigonometry

Trigonometry is the measurement of triangles, useful to find the lengths of the sides and the measurements of the angles of a triangle. We label the triangle as shown:

- "hypotenuse" – the longest side (which will be opposite the right angle). Abbreviated HYP
- θ – "theta", one of the angles other than the right angle
- "opposite" – the side straight across from θ. Abbreviated OPP
- "adjacent" - the side next to θ. Abbreviated ADJ



Trig uses three functions to show the relationships between the lengths of the sides and the measurements of the angles in the triangle. These are sine, cosine and tangent. The ratio of any two sides determines the measure of the angle θ.

$$\sin\theta = \frac{OPP}{HYP} \qquad \cos\theta = \frac{ADJ}{HYP} \qquad \tan\theta = \frac{OPP}{ADJ}$$

If you know two of the sides you can find θ by using the arcsine, arccosine and arctangent, usually written $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$

$$\theta = \sin^{-1}\left(\frac{OPP}{HYP}\right) \qquad \theta = \cos^{-1}\left(\frac{ADJ}{HYP}\right) \qquad \theta = \tan^{-1}\left(\frac{OPP}{ADJ}\right)$$

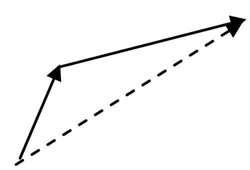
Vectors

Any measurement that has a number and direction can be expressed as a vector. For example if I drive North on I-77 at 70 mph, the vector would be $\vec{V} = 70\,mph\ North$, which is different than $\vec{V} = 70\,mph\ South$ or $\vec{V} = 76\,mph\ North$. The arrow notation is used to show that the quantity is a vector with both magnitude and direction. Here are some other vector quantities:

- Distance – walk 12 ft to the right $\quad \vec{s} = 12\ ft right$
- Acceleration – drop something , it speeds up at 9.8 m/s^2 down $\quad \vec{a} = 9.8\, m/s^2\ down$
- Force – push with 30 Newtons of force at an angle of 63^o $\quad \vec{F} = 30\ N \angle 63^O$

Vector Addition

Vectors can be added together. If I walk and then turn and go some more, how far away from my starting point and at what angle could I have travelled to reach the same spot. For any vectors A, B, C … N



Step 1: Add the x components to see how far right or left the vectors take you

$R_x = A_x + B_x + C_x + \ldots + N_x = A\cos\theta_A + B\cos\theta_B + C\cos\theta_C + \ldots + N\cos\theta_N$

Step 2: Add the y components to see how far up or down the vectors take you

$R_y = A_y + B_y + C_y + \ldots + N_y = A\sin\theta_A + B\sin\theta_B + C\sin\theta_C + \ldots + N\sin\theta_N$

Step 3: Find the length of the resultant to see how far from the starting point the vectors take you

$$R = \sqrt{{R_x}^2 + {R_y}^2}$$

Step 4: Find the angle to see the direction from the starting point

$$\varphi = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

Step 4a: Fix the angle depending on which quadrant the resultant lands in

1st quad: θ = φ $\quad$ 2nd Quad: θ = 180 – φ $\quad$ 3rd quad: θ = 180 + φ $\quad$ 4th Quad: θ = 360 - φ